10-1-1999

Strategic Buyers and Privately Observed Prices

Dirk Bergemann

Juuso Välimäki

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 1237

STRATEGIC BUYERS AND PRIVATELY OBSERVED PRICES

Dirk Bergemann and Juuso Välimäki

October 1999

Strategic Buyers and Privately Observed Prices*

Dirk Bergemann† Juuso Välimäki‡

August 1999

Abstract

A model of repeated price competition with large buyers is analyzed. The sellers are allowed to offer different prices to different buyers and the buyers act strategically. The set of subgame perfect equilibria is investigated under public and private monitoring.

With public monitoring the equilibrium set with large buyers *expands* relative to the standard model where each buyer is small and behaves myopically.

With private monitoring, where prices are not observable to the competing sellers, the set of equilibrium payoffs *shrinks*. In the finitely repeated game with private monitoring, all sales are made by the efficient seller. In the infinitely repeated game this result is preserved as long as the sellers condition their prices on the public history. In contrast to the finite horizon game, the set of pure strategy equilibria expands if the sellers are allowed to condition on their own past prices. Comparisons are drawn to Markovian equilibria of similar dynamic games.

*The authors gratefully acknowledge financial support from NSF Grant 9709887 and 9709340 respectively. The second author thanks the Helsinki School of Economics for its hospitality during a visit in the summer of 1999.

†Department of Economics, Yale University, New Haven, CT 06520-8268, dirk.bergemann@yale.edu.

‡Department of Economics, Northwestern University, Evanston, IL 60208-2009, U.S.A. and Department of Economics, University of Southampton, Southampton, UK, juuso.valimaki@soton.ac.uk

1 Introduction

This paper considers the informational role of past prices in a model of repeated price competition for large buyers. The basic model is a repeated extensive form game between two differentiated sellers and a single buyer with a unit demand in each period. At the beginning of each stage the sellers set prices, and the buyer selects the seller after having observed the prices. The competing sellers may or may not observe the prices offered by their competitors.

We describe the effects of informational constraints imposed on the repeated game. The case of public monitoring is compared with private monitoring where the sellers observe no signal of their opponent's price. In consequence, the identity of a deviator, seller or buyer, may not be common knowledge among the players.

We first show that with perfect monitoring, the set of subgame perfect equilibria is larger than in the standard model with small and hence myopic buyers. In fact, all individually rational payoffs are supportable as subgame perfect equilibria of the game. This is in contrast to the model with small buyers where only the set of *stage game* individually rational payoffs is supportable.

We then consider private monitoring. In finitely repeated games, sales are made only by the efficient seller in any sequential equilibrium of the game. Furthermore, bounds are obtained for the equilibrium payoffs to the buyer and the sellers in terms of the outside option that the buyer has in the form of the less efficient seller. In repeated games, all subgame perfect equilibria involve sales by the efficient seller only. In dynamic games (e.g. games with learning by doing on the part of the sellers), the myopically superior seller need not be the efficient intertemporal choice. We show that in dynamic games with a finite horizon, the sales in all sequential equilibria coincide with the efficient path of sales. Hence we observe that informational restrictions on the extensive form game have a similar effect as restricting attention to Markovian equilibria in the game.

To obtain some intuition into the results and the role of the strategic buyer, consider the incentives of the sellers and the buyer in a pure strategy equilibrium of a two period version of the model. In the second period, the sellers must price in such a way that the buyer is indifferent between the sellers. If any collusion in the first period is to be attempted, the sellers must have a method for punishing each other for any deviation in the second period. The only effective instrument for punishment is the coordination on a lower selling price off the equilibrium path than on the equilibrium path. Notice, however, that a lower price benefits the buyer to the same extent as it hurts the sellers. Since the intertemporal trade-offs between the sellers and the buyer balance out exactly, the efficient seller can always find a profitable price deviation that benefits the buyer as well at any node where an inefficient seller is supposed to make the sale. In Section 3, we show how this logic extends to all sequential equilibria of the finitely repeated game. The contrast to the game with a continuum of small identical buyers is immediate. If the buyers act based on their myopic payoffs only, then it is easy for the sellers to detect deviations by each other and as a consequence, the full individually rational payoff set is achievable in equilibrium whenever the products are differentiated and the division of surplus between the buyer and the efficient seller is not uniquely determined in the stage game subgame perfect equilibria. With multiple strategic buyers, more outcomes are feasible as well. The key observation here is that with many buyers, unexpected sales to a given buyer can be punished by lower future prices in the competition for all buyers. Hence the buyers' strategic motives for deviations can be minimized.

In the infinitely repeated version of the game with private monitoring, the set of pure strategy equilibrium payoffs is expanded by allowing the sellers to condition on their own past actions. In particular, we show that when the sellers charge prices that depend on their own prices, the entire individually rational set of payoffs is achievable.

In the infinite horizon model, a discrepancy arises between strategies based on public histories, which includes the sales history, and those based on private histories, which include the past prices. Since there is no last period, the buyer's indifference between the sellers can be broken down. In

Section 4, we show that the outcomes in the game between a single seller and the buyer are indeterminate in the infinite horizon model, when sellers and buyer condition their strategies on past prices. We use this result to obtain a folk theorem for the infinitely repeated game. The basic idea is to treat the periods when the two sellers are to make sales as strategically independent from each other and use the strategies from the one seller - one buyer model separately for both sellers. When current prices depend only on past public signals, i.e. the buyer's past purchasing decisions, the sellers cannot collude at all. In consequence all equilibria are efficient as are the Markovian equilibria. Hence we find out that in the infinite horizon model, the restriction to Markovian equilibria is payoff equivalent to a stronger informational restriction than in the finite horizon case.

A number of papers have analyzed the problem of collusion under imperfect monitoring. Starting with Green & Porter (1984), a popular approach has been one in which the actions taken by the sellers generate a publicly observable signal. All players may then condition their continuation play on this signal. This line of work culminates in the papers by Abreu, Pearce & Stacchetti (1990) and Fudenberg, Levine & Maskin (1994) where it is shown that as long as a minimal statistical requirement on the quality of the signal is satisfied, a collusive arrangement may be supported in a perfect public equilibrium. A number of recent papers including Sekiguchi (1997) and Bhaskar & Damme (1999) question the appropriateness of assuming the existence of public signals. If a common signal can be constructed through e.g. preplay communication, then Kandori & Matsushima (1998) and Compte (1998) show that collusive equilibria are possible in games with imperfect private monitoring while Ben-Porath & Kahneman (1996) obtain the result for pure private monitoring. Anh (1997) has considered repeated games with private monitoring where one player observes the moves of all other players. His stage games are, however, in normal form and therefore his results (as well as his motivation) are quite different from ours. The current paper is the first to our knowledge to analyze a repeated game with an extensive form stage game in the context of private monitoring.

2 Basic Model

Two sellers, $j \in \{1,2\}$ sell a product to a single buyer with unit demand repeatedly over time.[1] The maximum amount that the buyer is willing to pay for seller j's products is denoted by v^j for $j \in \{1,2\}$. At the beginning of period $t \in \{0,1,...\}$, the sellers announce simultaneously prices p_t^j and the buyer chooses between the sellers. The buyer's net payoff in period t is given by $v^j - p_t^j$ when buying from seller j. We normalize the production cost to zero so that the per period payoff to the seller is equal to the revenue. All players discount future with a common discount factor $\delta \in [0,1]$ and maximize the sum of their expected future payoffs. Notice that in our model, the payoffs to all parties are quasilinear in the payment, p_t, and as a result, Pareto efficiency coincides with surplus maximization.

3 Finite Horizon

3.1 Perfect Monitoring

Consider first the benchmark case of full information. In this case, secret price cuts are not possible as all players have perfect information about all previous moves by all previous players at each decision node (apart from the simultaneous pricing decisions, of course). Let T denote the length of the horizon in this game. When deciding her price in period $t \leq T$, seller j has observed history $h_t^j = \left\{p_0^1, p_0^2, d_0, ..., p_{t-1}^1, p_{t-1}^2, d_{t-1}\right\}$ where $d_t \in \{1,2,R\}$ is the buyer's choice of the seller in period t, and R denotes the rejection of both sellers' offers. In addition to the history available to the two sellers in period t, the buyer also knows the period t prices chosen by the two sellers and thus her information is given by $h_t^b = h_t^j \cup \left\{p_t^1, p_t^2\right\}$. The (behavior) strategies in period t available to seller j are then the functions from the set of possible period t histories, H_t^j to real numbers. The buyer's (behavior) strategies in period t are functions from all possible period t histories, H_t^b

[1] This restriction to just two sellers is without loss of generality in the current repeated game framework. It has implications in the more general dynamic game setting in section 3.

to $\{1, 2, R\}$. Let $H_t^p = H_t^1 \cap H_t^2$ denote the public history in period t.

To simplify notation, we assume that there is no discounting between the periods. The results we obtain hold for all δ sufficiently close to 1. Denote the $T-$fold repetition of the stage game by $\Gamma(T)$. Observe first that the stage game has a continuum of subgame perfect equilibrium payoffs if $v^1 \neq v^2$. Without loss of generality, let $v^1 > v^2$ and define $\Delta v = v^1 - v^2$.[2] Any pair of prices, (p^1, p^2) such that $0 \leq p^1 \leq \Delta v$, $p^2 = p^1 - \Delta v$, and buyer's strategy,

$$d = \begin{cases} 1 \text{ whenever } p^2 \geq p^1 - \Delta v \text{ and } p^1 \leq v^1, \\ 2 \text{ whenever } p^2 < p^1 - \Delta v \text{ and } p^1 \leq v^1, \\ R \text{ otherwise,} \end{cases} \tag{1}$$

is a subgame perfect equilibrium of this extensive form game. Denote the buyer's stage game strategy described above by d^*. Notice that in all stage game equilibria, seller 1 makes the sales, and as a consequence, seller 2 has a surplus of 0. Since we have multiple stage game payoff vectors, the continuation payoffs can be made dependent on the actions chosen, and there is a chance that a wide variety of outcomes might be supportable in subgame perfect equilibrium.

If the buyers are myopic, it is easy to show that the payoff vector in each stage must lie in

$$U = co\left\{(0,0,0), \left(v^1, 0, 0\right), \left(0, v^2, 0\right), \left(0, 0, v^1\right)\right\},$$

where $u = (u^1, u^2, u^b) \in U$. As a consequence, all subgame perfect equilibrium payoff vectors with myopic buyers must also lie in U.

The closure of the set of individually rational feasible payoff vectors in the stage game is, however, given by:

$$U^F = \{u \geq 0 \mid u^1 + u^2 + u^b \leq v^1\}.$$

Apart from dealing with an extensive form stage game, the stage game subgame perfect equilibria are not distinct in payoffs for seller 2. Hence we

[2] If $v^1 = v^2$, then the stage game has a single subgame perfect equilibrium payoff, and by backward induction, the finitely repeated game has a unique payoff vector as well.

cannot appeal directly to the result in Benoit & Krishna (1985) to conclude the existence of collusive equilibria in this game. Our first result shows that the set of equilibrium payoffs in $\Gamma(T)$ converges to U^F as $T \to \infty$. Strategies reminiscent to those in Benoit & Krishna (1985) are used to establish this.

Proposition 1 *Fix $u \in U^F$ and $\epsilon > 0$. Then there is a $\widehat{T}$ such that whenever $T \geq \widehat{T}$, $\Gamma(T)$ has a subgame perfect equilibrium (in pure strategies) with the average payoffs of the players in $B_\epsilon(u)$.*

P roof. We show how payoffs arbitrarily close to the vertices of U^F can be supported in SPE for T large enough. Since

$$U^F = co\{(0,0,0), (v^1,0,0), (0,v^1,0), (0,0,v^1)\},$$

the result follows upon observing that each $u \in U^F$ can be approximated by a convex combination of the vertices with rational coefficients. This corresponds to stacking together independent copies of the equilibria supporting the vertices in appropriate proportions.

(i) The stage game SPE profile $(p^1, p^2, d) = (0, -\Delta v, d^*)$ yields the payoff $(0, 0, v^1)$ and minmaxes both of the sellers. Playing $(0, -\Delta v, d^*)$ in all subtrees regardless of history is obviously a subgame perfect equilibrium. We can then also use this profile to punish both sellers for any deviations.

(ii) To find an equilibrium payoff close to $(v^1, 0, 0)$, consider the following strategies. In the last K periods, the play consist of $(p^1, p^2, d) = (\Delta v, 0, d^*)$. In the previous round, play $(p^1, p^2, d) = (v^2 + \Delta v + 1, v^2, d^*)$ and in the first $T-K-1$ periods, play $(p^1, p^2, d) = (v^1, v^1, d^*)$. Choose K to satisfy $K > \frac{v^1}{\Delta v}$. Any deviation by the buyer is ignored and any deviation by either of the firms leads to permanent reversion to $(p^1, p^2, d) = (0, -\Delta v, d^*)$. If T is chosen to be large enough, then the average payoffs to the players are approximately $(v^1, 0, 0)$.

(iii) For $(0, v^1, 0)$, let K be as before, and let the equilibrium actions in the first period be $((T-K-2)\,v^1 + \Delta v, (T-K-2)\,v^1, 2)$. In the next $T-K-1$ periods, $(p^1, p^2, d) = (0, -\Delta v, d^*)$ on the equilibrium path and in the last K periods, $(p^1, p^2, d) = (\Delta v, 0, d^*)$. Any deviation by either of the firms leads to the play of $(p^1, p^2, d) = (0, -\Delta v, d^*)$ until the end of the game and any deviation by the buyer in the first period leads to the play described in (ii).

(iv) Finally, strategies supporting a payoff close to $(0,0,0)$ are obtained by substituting $\left(v^2+\Delta v+1, v^2+1, d^*\right)$ for the first $T-K-1$ periods in (ii). Otherwise the description of the strategies is identical to (ii). ■

Since the result above is also valid for δ sufficiently close to 1, it is clear that the same set of equilibria can be supported in sequential equilibrium of the infinitely repeated version of the game.

3.2 Private Monitoring

In this section, we consider the game with private information about past moves. In particular, we assume that each p_t^j is observable to seller j and the buyer, but not to the other seller. The buyer's information sets are unchanged from the full information case, but the sellers have less information available: $h_t^j = \left\{p_0^j, d_0, ..., p_{t-1}^j, d_{t-1}\right\}$ for $j \in \{1,2\}$. Hence a price cut by one of the sellers is not directly observable to its competitor. In what follows, we look for sequential equilibria of this game. Notice also that the part of history that is common knowledge between the players is the sequence of purchases made by the buyer.

In order to analyze this game, it is useful to define a concept related to the correlated equilibria of the stage game. We maintain the requirement that the buyer's choice must be individually rational given the prices by the sellers, but we allow the sellers to correlate their choices of prices. Notice that the concept differs from the correlated equilibrium since a number of Nash equilibria of the stage game are excluded by the sequential rational behavior of the buyer.

Definition 1 *A SPE with correlation is a probability distribution* $F\left(p^1, p^2\right)$ *on* $\mathbb{R} \times \mathbb{R}$ *and a function* $d : \mathbb{R} \times \mathbb{R} \to \Delta\{1, 2, R\}$ *such that:*

1. *if* $d = d^*$ *as in (1);*

2. *for* $j \in \{1, 2\}$ *and for all* $p^j \in \mathrm{proj}_j \left(\mathrm{supp}\, F\left(p^1, p^2\right)\right)$:

$$p^j \in \arg\max_p p \Pr\{d(p, p^{-j}) = j\}.$$

Observe that in the first part of the definition, we are specifying the tie breaking rule to be in favor of the efficient seller. The following result would be unchanged if the buyer were to adopt any arbitrary tie-breaking rule.

Lemma 1 *Every SPE with correlation is payoff equivalent to a SPE in the stage game.*

P roof. Observe first that a price of 0 yields profit 0, and thus $p^j < 0$ and $p^j \in suppF\left(p^1, p^2\right)$ implies $\Pr\{v^j - p^j \leq v^{-j} - p^{-j}\} = 1$. Since this holds simultaneously for both sellers, we conclude, recalling $v^1 > v^2$, that in all correlated equilibria, $p^2 \geq -\Delta v$ and $p^1 \geq 0$.

By calculating the profit for seller 2, we conclude that $F^1\left(\acute{\Delta} v \mid p^2\right) < 1$ implies $p^2 > 0$. Hence for each $p^2 \in \mathrm{proj}_2 \left(\mathrm{supp}\, F\left(p^1, p^2\right)\right)$, either $p^2 \leq \min\{p^1 : \left(p^1, p^2\right) \in \mathrm{supp}\, F\left(p^1, p^2\right)\} - \Delta v$ or $p^2 > 0$. Notice that the first inequality states that it is not possible to have overlapping supports for p^2 for a set of positive (marginal) probability of p^1. Thus whenever $p^2 < 0$, we are (essentially) restricted to the "diagonal" $p^2 = p^1 - \Delta v$.

The case where $p^2 > 0$ with positive marginal probability is ruled out by usual undercutting arguments. The claim follows once we observe that every pair of prices on the"diagonal" $p^2 = p^1 - \Delta v$ is also a SPE of the stage game. ■

Another way of stating Lemma 1 is that the set of stage game equilibria is unchanged even if private signals are allowed for since all of the correlated equilibria are public randomizations of the Nash equilibria. Since privately observed histories generate private signals to the sellers, Lemma 1 shows that the private signals cannot be used in a nontrivial manner in the continuation play. The next proposition shows that restricting the observability in the game reduces dramatically the set of equilibrium outcomes.

Proposition 2 *With private monitoring, the set of sequential equilibrium average payoffs reduces to*

$$U^p = \left\{ \left(u^1, 0, u^b \right) \mid u^1 + u^b = v^1 \text{ and } 0 \leq u^1 \leq \Delta v \right\}.$$

In particular, all sequential equilibria are Pareto efficient.

P roof. The claim is proved by backwards induction. In the last period, regardless of history, a stage game correlated equilibrium must be played. Lemma 1 implies then that the last period purchase must be from seller 1 and therefore the last period payoff falls into the set U^p.

Assume next that in the last n periods, the continuation payoffs after all previous histories can be generated by using subgame perfect stage game strategies in the last n periods. By Lemma 1, we may write the continuation payoffs to the players as $\left(u^1(j), u^2(j), u^b(j)\right)$ for the buyer's decision $j \in \{1, 2, R\}$ in period $n+1$. By the induction hypothesis,

$$u^1(j) + u^b(j) = nv^1 \text{ and } 0 \leq u^1 \leq \Delta v.$$

Assume that seller 2 makes the sale at some node with $n+1$ periods left. Denote the private history of seller j consistent with reaching that node by h^j for $j \in \{1, 2\}$. At that node, $p^2\left(h^2\right) \geq 0$ as $u^2(2) = 0$ by the induction hypothesis. The buyer prefers seller 2 if

$$v^1 - p^1\left(h^1\right) + u^b(1) \leq v^2 - p^2\left(h^2\right) + u^b(2) \text{ or}$$

$$p^1\left(h^1\right) \geq v^1 - v^2 + p^2\left(h^2\right) - u^b(2) + u^b(1).$$

But using the induction hypothesis, this implies that the buyer prefers seller 2 if

$$p^1\left(h^1\right) \geq p^2\left(h^2\right) + v^1 - v^2 - u^1(1) + u^1(2), \text{ or}$$

$$p^1\left(h^1\right) + u^1(1) \geq p^2\left(h^2\right) + v^1 - v^2 + u^1(2).$$

Therefore, since $\Delta v > 0$ and $p^2\left(h^2\right) \geq 0$, seller 1 has a strict incentive to undercut and therefore the buyer must buy from the efficient seller in each period at a price not exceeding Δv.

Since all $u \in U^p$ can be supported as stage game subgame perfect equilibria, the claim follows. ■

The economic significance of this result is immediate. If the buyer prefers the product of seller 1 to the product of seller 2, then the equilibrium price of the efficient seller is between 0 and the quality difference in all periods, and the efficient seller makes all the sales in the model. As a result, we see that the possibilities for collusion are severely limited by the unobservability of opponents' prices in the finite horizon model.

The final proposition in this section extends the above result to the case where the stage game payoffs may be changing from period to period. We maintain the assumption of common knowledge of the payoffs between the players at all nodes of the game and as a result, we allow the stage game payoffs to depend only on the publicly observed history, i.e. the sequence of purchases by the buyer. The buyer has a valuation of $v^j\left(h_t^p\right)$ for the product of seller j after public history h_t^p. The changes in the valuations could be interpreted as resulting from learning by the buyer or from learning by doing by the sellers (in which case we would interpret v^j as the valuation net of the production cost).

Proposition 3 *In every finitely repeated dynamic game, the path of sales in all sequential equilibria coincides with the socially efficient path.*

P roof. The backwards induction argument in the previous proposition extends immediately to this case as well. ■

Notice that even though a restriction on the observability of past moves produces a reduction in the equilibrium set, the sales path does not collapse to the repetition of the myopic Nash equilibria. The sellers and the buyer can still take advantage of the changes in payoffs over time as the payoffs to the players conditional on the public history remain common knowledge.

3.3 Multiple Buyers

We conclude this section with an extension to $I \geq 2$ strategic buyers. The products are assumed to have the same value for all buyers. We assume that the sellers quote separate prices for the buyers and that the prices are not observable to outsiders, i.e. other buyers and other sellers. Denote seller j's price quoted to buyer i in period t by p_t^{ij} for $j \in \{1, 2\}$ and $i \in \{1, ..., I\}$. Denote the purchasing decision of buyer i in period t by d_t^i. The key difference to the single buyer case is that the buyers do not internalize their impact on the continuation values of the other buyers when contemplating a deviation. As a result, the earlier argument is weakened as not all future losses to a seller from breaking the collusive agreement are recorded as gains to the individual buyer. We demonstrate the effect of adding more buyers to the model by showing how the inefficient seller can achieve an increasing fraction of the total surplus as the number of buyers increase.

The equilibria we construct are of a very simple type. On the equilibrium path, the inefficient seller 2 sells at price v^2 in the first K periods and in the remaining $T-K$ periods, the efficient seller sells at price Δv. A deviation by any buyer (towards seller 1) in any of the first K periods results immediately in a switch to the subgame perfect equilibrium minmaxing both sellers. The (average) cost of such a deviation for seller 1 is given by

$$\frac{I\left(T-K\right)\left(v_1-v_2\right)}{T}$$

whereas the (average) gain for an individual buyer is bounded from above by

$$v_1 - \frac{T-K}{T} v_2.$$

The cost equals the gain if

$$\frac{K}{T} = \frac{Iv_1 - Iv_2 - v_1 + v_2}{Iv_1 - Iv_2 + v_2}, \tag{2}$$

and as a consequence, we can solve for the maximal number $K = K\left(T\right)$ that is consistent with an equilibrium of the form above.

Observe next that $K/T \rightarrow 1$ as $I \rightarrow \infty$, and as a consequence, we conclude that

$$\left(0, v_2, 0\right)$$

can be achieved as an average equilibrium payoff as $I \to \infty$. For any given I, we can support equilibrium average payoff (per buyer) vectors of the form:

$$\left(\frac{v_1 (v_1 - v_2)}{Iv_1 - Iv_2 + v_2}, \left(1 - \frac{v_1}{Iv_1 - Iv_2 + v_2}\right) v_2, \frac{v_1 v_2}{Iv_1 - Iv_2 + v_2}\right). \tag{3}$$

The next step is to use this equilibrium as a punishment for the buyers. Since the price of the seller who is not making sales in a given period can always be increased to the point where the buyer is indifferent between the two sellers, we can consider the action of rejecting both sellers as a deviation by the buyer. It is then possible to support average payoffs:

$$\left(\frac{v_1 (v_1 - v_2)}{Iv_1 - Iv_2 + v_2}, \left(1 - \frac{v_1}{Iv_1 - Iv_2 + v_2}\right) v_1, \frac{v_1 v_2}{Iv_1 - Iv_2 + v_2}\right)$$

in equilibrium. Notice that here the payoff for seller 2 converges to v_1 as $I \to \infty$.

The strategies supporting these payoffs are as follows. In the first $\sqrt{T}$ periods, seller 2 charges

$$\left(1 - \frac{v_1}{Iv_1 - Iv_2 + v_2}\right) v_1 \sqrt{T}$$

The buyers can be induced to purchase (even though it would be myopically better not to) by a threat of switching to the equilibrium leading to the payoffs in (3) if any buyer deviates. In the second phase,

$$K - \sqrt{T},$$

the buyers purchase from seller 1 at price 0 to recoup the losses and in the third phase, approximately of length $T - K$, the equilibrium leading to payoffs (3) is played. The idea behind the new equilibrium is that the length of time at which seller 2 sells is minimized so that the efficiency losses are minimized, but seller 2 can extract a large part of the surplus by charging initially high prices.

Using similar arguments as before, it is again possible to show that all the other vertices of U^F can also be approximated given that the number of buyers is sufficiently large and that there are sufficiently many periods in the game.

We summarize the discussion. Denote the average payoffs per buyer to the sellers in a game of length T by v_T^j for $j \in \{1, 2\}$ and the average payoff in a symmetric equilibrium to the buyer by u_T.

Proposition 4 *For any $u \in U^F$ and for any $\varepsilon > 0$, there are $\widehat{I} < \infty$ and $\widehat{T} < \infty$ such that whenever $I \geq \widehat{I}$ and $T \geq \widehat{T}$, there is a sequential equilibrium with $\left(v_T^1, v_T^2, u_T\right) \in B_\varepsilon(u)$.*

It should be remarked that the strategies employed in Proposition 4 are very sensitive to collusion among the buyers. By sharing the cost of a single deviation, the buyers can improve their payoffs dramatically in the game. Notice that we have not proved that the equilibria above support the maximal collusion in this game for finite I and T.

To address the issue of optimal punishment strategies, recall that the optimal punishment strategy of seller 2 towards seller 1 consists of reducing his payoff to 0 and increasing the payoff of every buyer to v^1. Since the buyers can, in effect, bribe seller 1 to deviate and since the originator of a deviation is not observable to the other seller, it follows that the optimal punishment play for seller 2 is the equilibrium which solves

$$\min_{(p^*, d^*)} \max_i \left\{V^1 + V^i\right\}. \tag{4}$$

We conjecture that the equilibrium leading to payoffs 3 solves this minimization problem.

4 Infinite Horizon

4.1 Single Seller

We consider first the relationship between a single seller and a single buyer. In this case, the game is one of perfect information, and the set of sequential equilibria coincides with the subgame perfect equilibria.

Proposition 5 *In the single seller game, the set of SPE payoffs converges to the convex hull of all individually rational payoffs as $\delta \to 1$.*

P roof. The proof is by construction. We first show that the entire Pareto-frontier can be achieved as $\delta \to 1$. Then we show that the Pareto frontier allows us to sustain arbitrarily inefficient equilibria. Consider first the following class of stationary equilibria (on the equilibrium path). Define

$$p_t^* = \begin{cases} p, & \text{if } h_{t-1} = \quad \left(p_{t-1}, d_{t-1}^*\right) \\ v, & \text{if } h_{t-1} = \left(p_{t-1}, d_{t-1} \neq d_{t-1}^*\right) \end{cases} \tag{5}$$

and

$$d_t^* = \begin{cases} y, & \text{if } p_t \leq p_t^* \\ n, & \text{if } p_t > p_t^* \end{cases} \tag{6}$$

for some $p \in [\varepsilon_\delta, 1]$, for some $\varepsilon_\delta > 0$, where $\varepsilon_\delta \to 0$ as $\delta \to 1$. Consider first the problem of the buyer. The yes decision is clearly sequentially rational. The no decision is sequentially rational for all $p' > p$ if

$$v - p < \frac{\delta\,(v-p)}{1-\delta},$$

or for all $\delta > \frac{1}{2}$. But then it follows that the seller's strategy (5) is sequentially rational as well. Based on (5) and (6), we next show that we can support a single period in which no sales occur.

Consider the candidate equilibrium strategy in period 0:

$$p_0^* = 0$$

and

$$d_0^* = \begin{cases} y & \text{if } p_0 < p_0^*, \\ n & \text{if } p_0 \geq p_0^*. \end{cases} \tag{7}$$

The equilibrium continuation strategy for the seller is

$$p_t^* = \begin{cases} p & \text{if } h_0 = \quad (p_0 \geq p_0^*,\ \ d_0 = n)\,, \\ \underline{p} < p & \text{if } h_0 = (p_0 < p_0^*,\ \ d_0 \in \{y, n\})\,, \\ \bar{p} > p & \text{if } h_0 = \quad (p_0 \geq p_0^*,\ \ d_0 = y). \end{cases} \tag{8}$$

and for the buyer, d_t^* coincides with (6).

It is easy to verify that d_0^* and p_0^* are sequentially rational given the continuations as long as $\underline{p}$ is chosen to be sufficiently close to 0 and $\bar{p}$ is chosen to be sufficiently close to v. To see how any pair of long run payoffs, $u = \left(u^1, u^b\right) \gg (0,0)$ such that $0 < u^1 + u^b \leq v$ can be supported in a subgame perfect equilibrium, write $u = \alpha\,(0,0) + (1-\alpha)\,(p, v-p)$ for some α and p with $0 \leq \alpha < 1,\ 0 < p < v$. For δ close enough to 1, there is a N such that δ^N is close to α, and the equilibrium can be constructed by using strategies d_0^* and p_0^* in the first N periods if there are no deviations and by reverting to (p_t^*, d_t^*) upon the first deviation. Since $0 < p < v$ and δ is close to 1, it is always possible to choose $\underline{p}$ and $\bar{p}$ to satisfy sequential rationality. ■

Proposition 5 implies in particular that the equilibria can be arbitrarily inefficient. Notice the important role that conditioning on past prices plays in the argument. It is easy to see that any equilibrium where the prices charged depend only on the choices by the buyer must be efficient. In the next subsection, we generalize this idea to show how any individually rational payoff can be supported in a pure strategy sequential equilibrium of the infinitely repeated game with two sellers.

4.2 Two Sellers

Next we consider, as in Section 3, two sellers with values v^1 and $v^2 < v^1$. The analysis in Proposition 5 then suggests the following structure for individually rational payoffs that can be sustained in equilibrium by the buyer and seller 1 while keeping $V^2 = 0$ in all continuations.

By ignoring seller 2 altogether, any continuation payoffs which have the following properties

$$V^B \geq \frac{v^2}{1-\delta},\ V^1 \geq 0$$

and

$$V^B + V^1 \leq \frac{v^1}{1-\delta} \tag{9}$$

can be implemented as a sequential equilibrium while maintaining $V^2 = 0$. In fact, a much larger set of payoffs is achievable in sequential equilibrium in pure strategies. The easiest way to see this is to notice that the two sellers can be treated separately from each other by using strategies of the type displayed in Proposition 5. It is useful to observe that the following result does not make use of the fact that a seller can distinguish between the cases where a buyer did not buy at all and where the buyer bought from the opponent.

Proposition 6 *In the game with two sellers, any payoff in U can be approximated by a sequential equilibrium payoff as $\delta \to 1$.*

P roof. Consider an arbitrary payoff $u \in \operatorname{int} U$. Write

$$u = \alpha\left(v^1 - p^1, p^1, 0\right) + \gamma\left(v^2 - p^2, 0, p^2\right) + (1 - \alpha - \gamma)(0, 0, 0)$$

for some $p^1, p^2 \geq 0$, $p^2 \leq v^2$and $0 \leq \alpha, \gamma < 1$, $\alpha + \gamma \leq 1$.

(i) Consider first $p^1 < \Delta v$. The equilibrium path consists of three phases. For N^n periods, both sellers quote prices $p^1 = p^2 = v^1$ and the buyer does not buy. For the following N^2 periods, seller 2 sells at price p^2 and seller 1 quotes a price of v^1. In the remaining periods, seller 2 quotes a price of v^1 while seller 0 sells at p^1. Any failure to purchase by the buyer on the equilibrium path results in the equilibrium seller raising the price as in Proposition 5. Deviations by the sellers result in lower continuation prices as in Proposition 5. For $\delta \approx 1$, N^n and N^2 can be chosen to match the prespecified α and γ.

(ii) Consider next $p^1 \geq \Delta v$. This case is similar to (i), but to maintain proper incentives, the first selling phase must involve sales by seller 1 at prices above p^1. In the second selling phase, seller 2 sells at p^2 and in the third, seller 0 sells at $p^1 < \Delta v$. Again, the incentives can be maintained by using strategies similar to the one in Proposition 5 and the lengths of the phases can be chosen to match α and γ. ∎

4.3 Equilibria in Public Strategies

In this subsection, we show that the equilibrium payoffs in the infinitely repeated game reduce to the payoffs on the efficient frontier when the sellers do not condition on their own private prices.

Proposition 7 *If $p_t^j : H_t^p \to \Delta(R)$, then the set of sequential equilibrium payoffs in the infinitely repeated game is U^p.*

P roof. The set of sequential equilibrium payoffs contains U^p since all $u \in U^p$ are stage game subgame perfect equilibrium payoffs. To prove the converse, note first that in any sequential equilibrium with a payoff vector not contained in U^p, the buyer must buy from seller 2 infinitely often. If not, we could use the backwards induction argument of the finite horizon section to derive a contradiction. It is also useful to note that each sequential equilibrium of the game is payoff equivalent to a pure strategy equilibrium as long as we restrict the strategies to depend on the public history alone.

Since we are dealing with sequential equilibria where the sellers use public strategies in this case, dynamic programming can be used readily as the seller's strategies condition on the public history only. Denote the value functions of the players by $\left(V^1(h^p), V^2(h^p), V^b(h^p)\right)$ and the continuation payoffs to the players by $(V^1(h^p, j), V^2(h^p, j), V^b(h^p, j))$ if the buyer purchases from seller j after public history h^p. The equilibrium conditions for seller 1 to make sales after history h^p are then stated as :

$$v^1 - p^1(h^p) + \delta V^b(h^p, 1) = v^2 - p^2(h^p) + \delta V^b(h^p, 2),$$

or

$$v^j - p^j(h^p) + \delta V^b(h^p, j) \leq 0 \text{ for } j = 1, 2$$

and

$$p^1(h^p) + \delta V^1(h^p, 1) \geq \delta V^1(h^p, 2)$$

as well as

$$\delta V^2(h^p, 1) \geq p^2(h^p) + \delta V^2(h^p, 2).$$

Since we have $v^1 - p^1(h^p) + \delta V^b(h^p, 1) = v^2 - p^2(h^p) + \delta V^b(h^p, 2)$ in all periods, then we can simply sum up the three equations above to get for all h^p :

$$W(h^p) = v^1 + \delta W(h^p, 1) \geq v^2 + \delta W(h^p, 2),$$

where $W(\cdot) = V^1(\cdot) + V^2(\cdot) + V^b(\cdot)$. But this implies that the sum of the equilibrium value functions of the players satisfies the recursive equation for the planner's optimization problem and as a result, we conclude that the equilibrium path must coincide with the planners optimal path.

It is easy to see that the buyer must be getting a strictly positive surplus in all subgames. Therefore, the only task remaining is to verify the claim on prices. This follows also from standard arguments. ■

The argument here is easily modified to allow v^j to depend on h^p. Again, the analogous result in the dynamic game case is that all the sequential equilibria where the sellers condition their prices only on the buyer's choices must be efficient. In light of the results that we have previously obtained on the efficiency of Markov Perfect Equilibria in closely related games (see Bergemann & Välimäki (1996)), this suggests that the informational restrictions on the full information model that are needed to reduce the set of equilibrium payoffs to that corresponding to the MPE are stronger in the case of infinitely repeated games than in finitely repeated games.

5 Conclusion

This paper analyzed a repeated pricing game with private monitoring. In the finite horizon model, the presence of a strategic buyer reduced the ability of the sellers to sustain collusion. Every punishment strategy which lowers the payoff of the deviating seller leads to an equal increase in the payoff of the buyer and vice versa. As private monitoring prevents the detection of the identity of the deviating player, collusion and inefficient allocations become impossible to sustain. With multiple buyers, this logic becomes weaker as the buyers fail to take into account the changes in the future payoffs of the other buyers caused by their own deviations.

Interestingly, in the infinite horizon model pure strategies based on the sales history and strategies including the private price history lead to a different set of equilibria. The discrepancy is due to the extensive form of the stage game introduced through the presence of a strategic buyer. We recall that in a normal form repeated games the two sets of pure strategies are known to induce the same set of equilibrium payoffs. The divergence between private and public strategies might be of interest also in other settings with a sequential structure such as repeated principal agent games with private monitoring.

References

Abreu, D., D. Pearce & E. Stacchetti. 1990. "Towards a Theory of Discounted Repeated Games with Imperfect Monitoring." *Econometrica* 58:1041–1068.

Anh, I. 1997. "Private Monitoring in Repeated Games.".

Ben-Porath, E. & M. Kahneman. 1996. "Communication in Repeated Games with Private Monitoring." *Journal of Economic Theory* 70:281–297.

Benoit, J.-P. & V. Krishna. 1985. "Finitely Repeated Games." *Econometrica* 53:905–922.

Bergemann, D. & J. Välimäki. 1996. "Learning and Strategic Pricing." *Econometrica* 64:1125–49.

Bhaskar, V. & E. Van Damme. 1999. "Moral Hazard and Private Monitoring.".

Compte, O. 1998. "Communication in Repeated Games with Imperfect Private Monitoring." *Econometrica* 66:597–626.

Fudenberg, D., D. Levine & E. Maskin. 1994. "The Folk Theorem with Imperfect Public Information." *Econometrica* 62:997–1040.

Green, E. & R. Porter. 1984. "Noncooperative Collusion under Imperfect Price Information." *Econometrica* 52:87–100.

Kandori, M. & H. Matsushima. 1998. "Private Observation, Communication and Collusion." *Econometrica* 66:627–652.

Sekiguchi, T. 1997. "Efficiency in Repeated Prisoner's Dilemma with Private Monitoring." *Journal of Economic Theory* 76:345–361.